CERN-TH/95-119

$k_\perp$ FACTORIZATION vs. RENORMALIZATION GROUP: A SMALL-x CONSISTENCY ARGUMENT [*)]

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Abstract

I investigate, at leading twist level, the consistency of the BFKL equation with the renormalization group, when next-to-leading log-x terms are included in the quark-sea channel of the anomalous dimension matrix. By use of $k_\perp$-factorization, I find that, besides next-to-leading small x resummation formulae, a leading, x-dependent redefinition of initial quarks and gluons is needed. Its interpretation and phenomenological relevance are briefly discussed.

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*) Work supported in part by E.C. "Human Capital and Mobility" contract # ERBCHRXCT 930357, and by M.U.R.S.T. (Italy).

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CERN-TH/95-119
May 1995

Recent experiments [1] on deep inelastic scattering (DIS) at small Bjorken x call for a better understanding of the theoretical predictions on the proton structure functions, in order to yield, hopefully, some unambiguous explanation of their observed small-x rise and of the related scaling violations.

Although present approaches to fit the HERA data are mostly based on QCD [2] and most of them successful, different authors follow different lines of thought and different procedures, according to whether they emphasize the perturbative Q^2-evolution based on the GLAP equations [3], or instead the high-energy, small-x evolution based on the BFKL equation [4].

On the other hand, the systematic use of $k_\perp$-factorization [5],[6], in order to combine the BFKL equation with the renormalization group, has shown that the high-energy structure implies resummation formulas for coefficient functions [5] and anomalous dimensions [7] in the effective small-x coupling $\alpha_s \log x$. This suggests that the two points of view mentioned before may indeed be equivalent, with the important – and fruitful – consequence that perturbation theory is recast in a resummed form, so as to explain the HERA data [8].

That is not all, however, because it is usually thought that the BFKL equation (and generalizations of it) are able to predict, for large enough scales, the small-x behaviour of the cross-section, i.e., the "perturbative Pomeron", even when there is no Q^2 evolution at all [9], and thus the anomalous dimension is not really relevant. Is this a sign that there is some essential difference?

The purpose of this note is to push further the consistency point of view in experiments with one large scale Q^2 (of DIS or Drell-Yan type) and at leading twist level, i.e., by neglecting subleading powers of Q_0^2/Q^2, where $Q_0 > \Lambda$ defines the boundary of the perturbative approach ($\alpha_s(Q_0^2) \lesssim 1$).

By analysing the BFKL Green's functions with quark loops up to next-to-leading (NL) $\log x$ level, I will show that indeed they are equivalent to the GLAP ones with properly resummed anomalous dimensions, but they also involve leading, x-dependent redefinition of initial quarks and gluons. It will appear that such a redefinition carries the small-x evolution at fixed scale mentioned before, even if its precise form is questionable in a perturbative approach.

An interesting by-product of the above analysis will be a simple calculation of the (subleading) quark anomalous dimensions $\gamma_{qg}^N, \gamma_{qq}^N$ in a slightly different factorization scheme from the ones of MS-type used by Catani and Hautmann [7].

I will consider, for a given moment index N, the Green's function matrix $G_{ba}^N(Q^2, Q_0^2)$ of the BFKL equation in the flavour singlet sector defined by

$$G_{ba}^N(Q^2, Q_0^2) = \int_0^{Q^2} d^2k \;\; \mathcal{F}_{ba}^N(\mathbf{k}, Q_0^2) \;, \quad (a, b = q, g) \;, \tag{1}$$

where the $\mathcal{F}_{ba}$'s are unintegrated structure functions of parton b in parton a, satisfying small-x equations of the BFKL type, to be specified shortly.

In the gluon channel, I shall take $\mathcal{F}_{gg}^N = \mathcal{F}_N(\mathbf{k}, Q_0)$, satisfying the usual BFKL equation [10]

in four dimensions, with fixed initial gluon virtuality $k_a^2 = Q_0^2$, i.e.,

$$\mathcal{F}_N(\mathbf{k}, Q_0) = \delta(\mathbf{k}^2 - Q_0^2) + \frac{\bar{\alpha}_s}{N-1} \int \frac{d^2q}{\pi\mathbf{q}^2} \left[\mathcal{F}_N(\mathbf{k}+\mathbf{q}, Q_0) - \Theta(k-q)\, \mathcal{F}_N(\mathbf{k}, Q_0)\right] \tag{2}$$

where $\bar{\alpha}_s = \frac{3\alpha_s}{\pi}$ is the strong coupling constant.

The solution of Eq. (2) admits an integral representation in the anomalous dimension plane which, inserted in the definition (1), yields

$$G_{gg}^N = \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} \frac{d\gamma}{2\pi i} \frac{(Q^2/Q_0^2)^\gamma}{\gamma(1-\frac{\bar{\alpha}_s}{N-1}\,\chi(\gamma))^{-1}} \,, \tag{3}$$

where the eigenvalue function

$$\chi(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma) \tag{4}$$

implicitly defines the perturbative branch of the gluon anomalous dimension by

$$1 = \frac{\bar{\alpha}_s}{N-1}\, \chi(\gamma) \,, \tag{5a}$$

$$\gamma_{gg}^N = \gamma_N(\alpha_s) = \frac{\bar{\alpha}_s}{N-1} + 2\zeta(3)\left(\frac{\bar{\alpha}_s}{N-1}\right)^4 + \ldots \quad . \tag{5b}$$

For $Q^2 > Q_0^2$, G^N is determined by the γ poles given by (5a) in the left-hand γ plane and, at leading twist level, by γ_N, in the form

$$G_{gg}^N = K_N(\alpha_s) \left(\frac{Q^2}{Q_0^2}\right)^{\gamma_N(\alpha_s)} \left(1 + O\left(\frac{Q_0^2}{Q^2}\right)^p\right) \tag{6}$$

where the coefficient

$$K_N(\alpha_s) = \left[-\gamma_N \frac{\bar{\alpha}_s}{N-1}\, \chi'(\gamma_N)\right]^{-1} = 1 + 6\zeta(3)\left(\frac{\bar{\alpha}_s}{N-1}\right)^3 + \ldots \tag{7}$$

will be further discussed in the following.

Note that, at this stage, I keep α_s frozen, because the running of α_s mixes with other NL log effects, which are still under investigation [11]. Of course, once a factorization of the R.G. type is established, the running of α_s will be restored.

I shall consider, instead, NL effects due to quark loops, because they couple directly to the photon. The final quark channel is in fact defined in terms of the structure function F_2 [1]

$$F_{2a}^N = \sum_f e_f^2\; G_{qa}^N \tag{8}$$

[1] This was called the DIS scheme in Ref. [7]. However, the present definition of gluons is different [cf. Eq. (18)].

and, by $k_\perp$ factorization, its evolution is given [7] in terms of the off-shell $\gamma g \to \bar{q}q$ cross-section $\hat{\sigma}_2$ (Fig. 1)

$$\frac{\partial}{\partial \log Q^2} \; G_{qa}^N \; (Q^2, Q_0^2) = \int \; d^2k \; \frac{\partial}{\partial \log Q^2} \; \hat{\sigma}_2 \left(\frac{k^2}{Q^2}\right) \; \mathcal{F}_{ga}(k^2, Q_0^2) \; , \tag{9}$$

where the lowest-order expression of $\hat{\sigma}_2$ was given in Ref. [5].

Note that the definition (1) and Eq. (9) carry $k_\perp$- integrations up to $k^2 = 0$. Nevertheless, for fixed α_s, they are well defined, because, according to Eq. (3), the region $k^2 < Q_0^2$ is governed by the r.h. plane anomalous dimensions $(1 - \gamma_N)$ and higher, and is hence automatically suppressed. It appears, therefore, that fixing the initial parton virtuality at Q_0 automatically regulates all intermediate collinear singularities, unlike what happens in (on-shell) dimensional subtraction schemes. Furthermore, the overall collinear singularity will be factorized in Q_0-dependent powers, as done in Eq. (6) in the gluon channel. I will call this factorization procedure the Q_0-scheme, to distinguish it from the ones of MS-type, used in Ref. [7].

The most important consequence of introducing quark loops comes from Eq. (9), where the explicit expression of $\hat{\sigma}_2$ [5] was used in Ref. [7] to provide the NL resummed formula for the off-diagonal anomalous dimension γ_{qg}. In fact, by setting $a = g$ and by performing the $k_\perp$-integration with the help of Eq. (3), one gets

$$\frac{\partial}{\partial \log Q^2} \; G_{qg}^N \; (Q^2, Q_0^2) \equiv \dot{G}_{qg}^N = 2N_f h_2(\gamma_N(\alpha_s)) \;\; (G_{gg}^N + O(Q_0^2/Q^2)^p) \tag{10a}$$

where

$$h_2(\gamma) = \frac{\alpha_s}{3\pi} \; \frac{1 + \frac{3}{2}\gamma(1-\gamma)}{1 - \frac{2}{3}\gamma} \; \frac{[\Gamma(1-\gamma)\Gamma(1+\gamma)]^3}{\Gamma(2-2\gamma)\Gamma(2+2\gamma]} \; , \tag{10b}$$

and N_f is the number of quark flavours. On the other hand, the customary GLAP evolution equations would give, to NL accuracy [12]:

$$\dot{G}_{qg}^N = \gamma_{qa}^N \; G_{ag}^N = \gamma_{qg}^N \; G_{gg}^N + O(\alpha_s^2) \; , \tag{11a}$$

$$\dot{G}_{qq}^N = \gamma_{qa}^N \; G_{aq}^N = \gamma_{qg}^N \; G_{gq}^N + \gamma_{qq}^N + O(\alpha_s^2) \; . \tag{11b}$$

Therefore, by comparing (11a) with (10a), we obtain

$$\gamma_{qg}^N(\alpha_s) = 2N_f \;\, h_2(\gamma_N(\alpha_s)) \; . \tag{12}$$

This determination of γ_{qg} in the Q_0 scheme differs from Ref. [7] by the absence of the so-called R_N factor [analogous to K_N in Eq. (6)], which arises [13] from regularizing and factorizing the intermediate collinear singularities in MS-type schemes. I shall further comment on this point later on.

By then introducing initial quarks (see Fig. 1), and by neglecting internal quark loops at NL level, I obtain integral representations for all G_{ab}'s, as follows

$$\dot{G}_{ab}^N = \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} \; \frac{d\gamma}{2\pi i} \; \frac{(Q^2/Q_0^2)^\gamma}{1 - \frac{\bar{\alpha}_s}{N-1}\chi(\gamma)} \;\; h_a(\gamma) \; k_b(\gamma) \; , \tag{13a}$$

where

$$h_g = k_g = 1, \quad h_q = \frac{2N_f}{\gamma}\; h_2(\gamma)\;, \quad k_q = \frac{C_F}{C_A}\;\frac{1}{\gamma}\;\frac{\bar{\alpha}_s}{N-1}\;, \tag{13b}$$

with $C_F = 4/3$ and $C_A = 3$.

It is now not difficult to evaluate Eq. (13) for $Q^2 \gg Q_0^2$, at leading twist level and NL $\log x$ accuracy. Since the flavour singlet anomalous dimension matrix has two eigenvalues, one close to γ_N, and the other next-to-leading, I will keep in Eq. (13) all terms coming from poles at $\gamma = \gamma_N$ and $\gamma = 0$ which, after some algebra, yield the following results.

(a) The G_{ab}'s *do satisfy* the GLAP-type equations (11), i.e., in matrix form,

$$\dot{G}_N(Q^2, Q_0^2) = \Gamma_N(\alpha_s) G_N(Q^2, Q_0^2) \tag{14}$$

with γ_{gg} (γ_{qg}) given in Eq. (5) [Eq. (12)] respectively, and

$$\gamma_{gq}^N = \frac{C_F}{C_A}\;\gamma_N\;, \quad \gamma_{qq}^N = \frac{C_F}{C_A}\;\left(\gamma_{qg}^N - \gamma_{qg}^{(1)}\right)\;, \tag{15}$$

where $\gamma_{qg}^{(1)} = 2N_f\alpha_s/3\pi$ is the lowest-order expression.

Equations (5), (12) and (15) yield the promised resummation formulas in this scheme.

(b) The normalization of the G_{ab}'s *is not* the same as the corresponding GLAP Green's function matrix, which for frozen α_s is $F_N = (Q^2/Q_0^2)^{\Gamma_N}$, but differs from it by a constant, finite, N-dependent matrix K^N

$$G_N(Q^2, Q_0^2) = (Q^2/Q_0^2)^{\Gamma_N}\; K^N(\alpha_s)\;, \tag{16}$$

where

$$\begin{aligned} K_{gg}^N &= K_N\;, \quad K_{gq}^N = \frac{C_F}{C_A}\;\left(\frac{\gamma_N^{(1)}}{\gamma_N}\;K_N - 1\right)\;, \\ K_{qg}^N &= \frac{\gamma_{qg}^N}{\gamma_N}\;K_N - \frac{\gamma_{qg}^{(1)}}{\gamma_N^{(1)}}\;, \quad K_{qq}^N = 1 + \frac{C_F}{C_A}\;\left(\frac{\gamma_N^{(1)}\gamma_{qg}^N}{\gamma_N^2}\;K_N - \frac{13}{6}\;\gamma_{qg}^{(1)}\right)\;, \end{aligned} \tag{17}$$

and $\gamma_N^{(1)} \equiv \bar{\alpha}_s/(N-1)$.

Several comments are in order. First, one should not be too surprised to find a different determination of NL anomalous dimensions in the Q_0 scheme. It is easy to check that $\Gamma_N^{(Q_0)}$ differs from $\Gamma_N^{(DIS)}$ in Ref. [7] by a redefinition of the gluon density, as follows:

$$\Gamma_N^{(Q_0)} = a_N^{\Gamma_L}\;\Gamma_N^{(DIS)}\;a_N^{-\Gamma_L}\;, \tag{18}$$

where Γ_L is the common leading part (provided by γ_{gg} and γ_{gq}) and a_N is a (properly chosen) scale-changing parameter.

It is, however, of some interest that the present determination does not include the R_N (or K_N) factors, which contain a strong Pomeron singularity (for instance, K_N diverges like $(\gamma_N - 1/2)^{-1}$ for $N \to N_{\mathbf{P}}$, or $\gamma_N \to 1/2$). For this reason, the resummed expression for $\Gamma^{(Q_0)}$ [Eq. (12)] is expected to be smoother than the one for $\Gamma^{(DIS)}$, and may be, as such, more palatable. A firm conclusion will be possible when NL contributions to γ_{gg}^N [which are changed by Eq. (18)] will be available.

More importantly, Eq. (16) clarifies that, if the BFKL equation is to provide the same parton densities $f_a(Q^2)$ as GLAP's at the scale Q^2, then the initial densities must be *different*, as follows

$$f_a^{\mathrm{GLAP}}(Q_0^2) = K_{ab}^N(\alpha_s)\ f_b^{\mathrm{BFKL}}(Q_0^2)\ . \tag{19}$$

The normalization matrix K^N carries the information related to the small-x evolution at the scale Q_0, and in fact contains the strong Pomeron singularity of the K_N factors mentioned before.

Having checked the evolution equations (11), the running coupling can be restored in the factorized formula (16), in the normal way

$$G^N = \exp\left(\int_{\log\frac{Q_0^2}{\Lambda^2}}^{\log\frac{Q^2}{\Lambda^2}} dt\ \Gamma_N(\alpha_s(t))\right)\quad K^N(\alpha_s(Q_0^2))\ , \tag{20}$$

with the usual uncertainty connected with NL terms in the gluon channel. In this interpretation, the matrix K^N depends on the strong coupling at the lower scale, and thus its precise form (17) will be affected by higher-twist unitarity effects [14] if Λ^2/Q_0^2 is not too small, and anyway much before they become relevant at the scale Q^2. Whatever the actual form of K^N, Eq. (20) teaches us that approaches based on the BFKL (resp. GLAP) equations, can only be compared when such redefinition of quarks and gluons is accounted for.

To conclude, the information coming from the BFKL equation for single hard scale processes can be incorporated in the renormalization group in a consistent picture. The effects of small-x evolution due to fluctuations in $k_\perp$ around Q_0 are factorized out and can be incorporated in a change of initial conditions. Whether such an "initial Pomeron" looks like the BFKL one or like the one of soft hadronic physics [15] is a (Q_0-dependent) question open to investigation [14], [16], which has not been touched here.

Acknowledgements

It is a pleasure to thank Stefano Catani and Francesco Hautmann for interesting discussions and suggestions, and Guido Altarelli, Keith Ellis, Stefano Forte, Giuseppe Marchesini, Paolo Nason and Graham Ross for helpful discussions.

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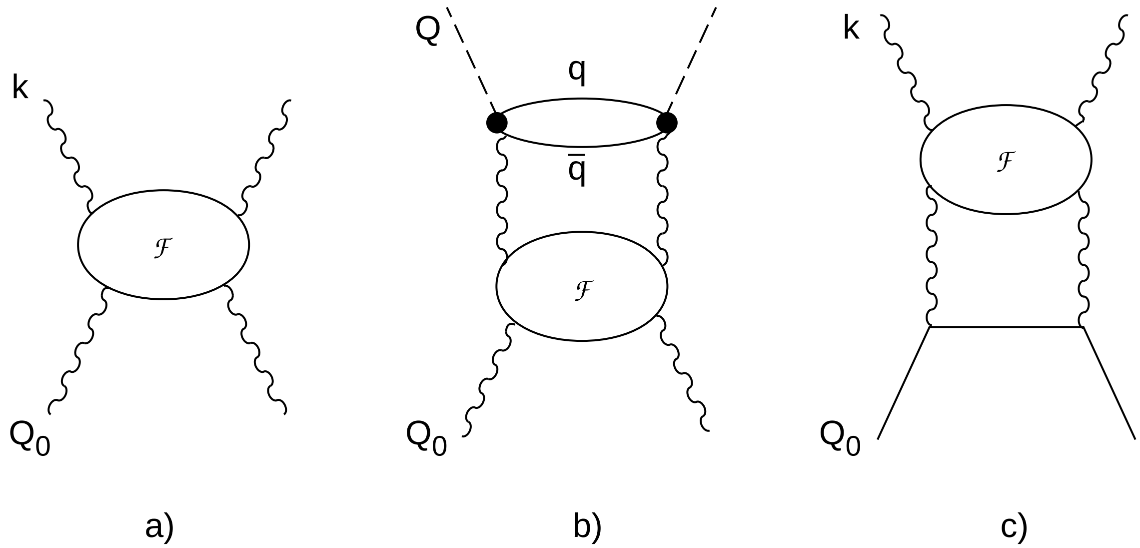

Fig. 1

Diagrammatic definition of (a) the gluon-gluon (b) the quark-gluon and (c) the gluon-quark Green's functions up to next-to-leading log x level. Wavy (dashed) lines denote (Regge) gluon (photon) exchanges.